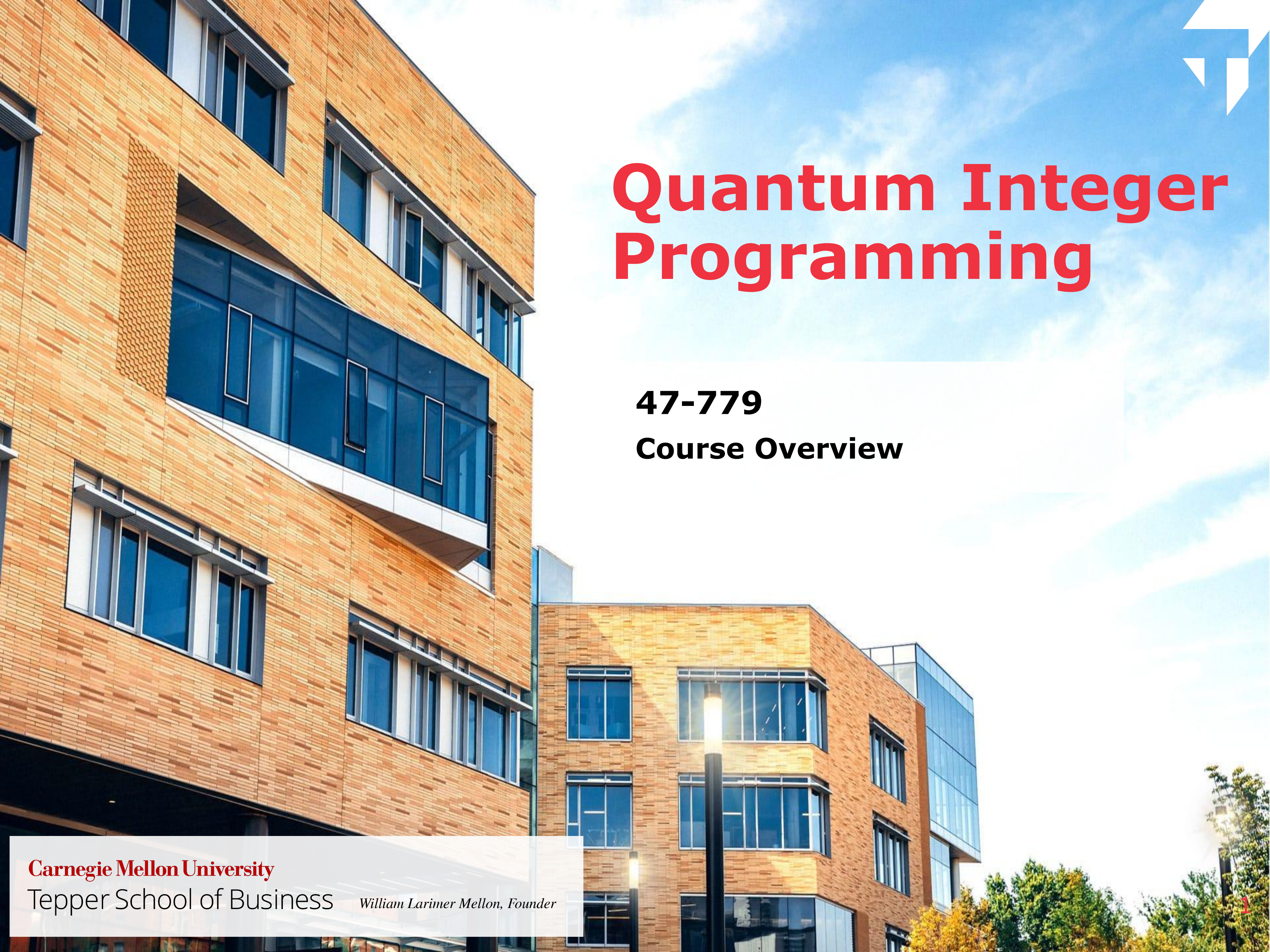


Quantum Integer Programming

47-779

Course Overview

47-779. Quantum Integer Programming. Fall 2020 - Mini 1

- Lecturers
- Objectives
- Expectations
- Pre-requisites
- Tentative Course Outline
- Grading Policy
  - Project choices and examples
- Course Policy
- USRA Collaboration

Lecturers

- Prof. Sridhar Tayur
  - Ford Distinguished Research Chair; University Professor of Operations Management; Tepper School of Business CMU
  - Academic Capitalist

- Dr. Davide Venturelli
  - Associate Director for Quantum Computing of the Research Institute of Advanced Computer Science (RIACS) at the USRA.
  - Senior Scientist NASA Quantum AI Laboratory

- David E. Bernal
  - Ph.D. Candidate at Chemical Engineering Department, CMU
  - 2019 awardee of USRA Feynman Quantum Academy Program at NASA Ames Research Center

Objectives

This course is primarily designed for graduate students and advanced undergraduates **interested in integer programming** and the **potential** of **near-term quantum** and **quantum-inspired computing** for solving **combinatorial optimization problems**.

- Appreciate the current status of quantum computing and its potential use for integer programming
- Access and use quantum computing resources (such as D-Wave Quantum Annealers, IBM/Google/Rigetti computers)
- Set up a given integer program to be decomposed and partially solved with quantum computing or other physics-based processors
- Work in groups collaboratively on a state-of-the-art project involving applications of quantum computing and integer programming

Expectations

- This course is not going to focus on the following topics:
  - Quantum Gates and Circuits
    - 15-459 Quantum computation in CS
  - Computational complexity theory
    - 15-651 Algorithm Design and Analysis in CS
  - Quantum Information Theory
    - 33-658 Quantum computation and Info theory in Physics
  - Analysis of speedup using differential geometry, algebraic topology, etc.
    - 21-752 Algebraic Topology or 21-759 Differential Geometry in Mathematics
-

Pre-requisites

- No explicit pre-requisites are listed but we recommend:
  - An **undergraduate-level** understanding of **probability, calculus, statistics, graph theory, algorithms,** and **linear algebra** is assumed.
  - Knowledge of **linear and integer programming will be useful**.
  - **Programming skills** are **strongly recommended** (Python preferred)
  - **Basic** concepts in **physics** are **recommended** but lack of prior knowledge is not an issue as pertinent ones will be covered in the lectures.
  - **No particular knowledge in quantum mechanics** or algebraic geometry **is required**.

Tentative Course Outline

- Integer Programming
  - Cutting plane methods
  - Test-set methods: Groebner and Graver Basis
- Ising model, Quadratic Unconstrained Binary Optimization (QUBO)
  - Simulated Annealing
  - Benchmarking classical methods
  - Formulating combinatorial optimization problems as QUBO
- GAMA: Graver Augmented Multiseed Algorithm
- Quantum methods for solving Ising/QUBO
  - Adiabatic Quantum Computation: Quantum Annealing
  - Gate-based Quantum Computing: Quantum Approximate Optimization Algorithm
- Specialized Hardware for solving Ising/QUBO
  - GPUs, TPUs
  - Digital Annealers
  - Oscillator based Computers
  - Coherent Ising Machines

Integer Programming

Current status and perspectives

Classical methods

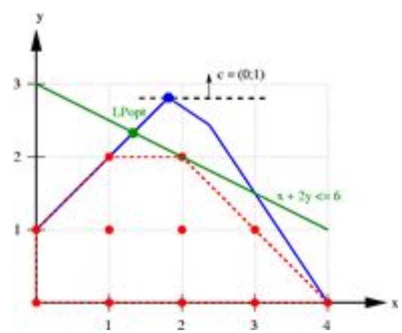
Methods based on divide-and-conquer

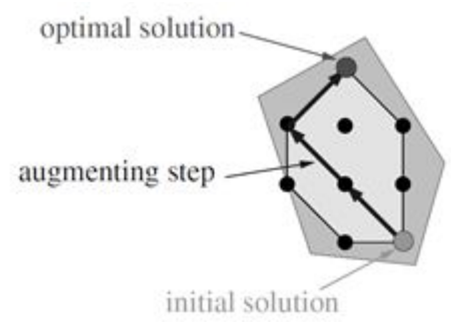
- Branch-and-Bound algorithms
- Harness advances in polyhedral theory
- With global optimality guarantees
- Very efficient codes available
- Exponential complexity

Not very popular classical methods

Methods based on test-sets

- Algorithms based on “augmentation”
- Use tools from algebraic geometry
- Global convergence guarantees
- Very few implementations out there
- Polynomial **oracle** complexity **once we have test-set**

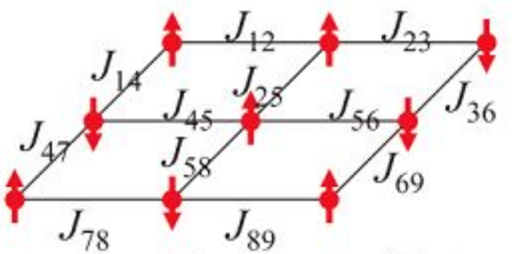




[1] https://de.wikipedia.org/wiki/Branch-and-Cut
[2] Algebraic And geometric ideas in the theory of discrete optimization. De Loera, Hemmecke, Köppe. 2012

Ising model, QUBO

Mental model and applications



$$H = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i\sigma_j - \sum_j h_j\sigma_j$$

$$\sum_{s_1=0}^{1}\sum_{s_2=0}^{1}\cdots\sum_{s_n=0}^{1}$$

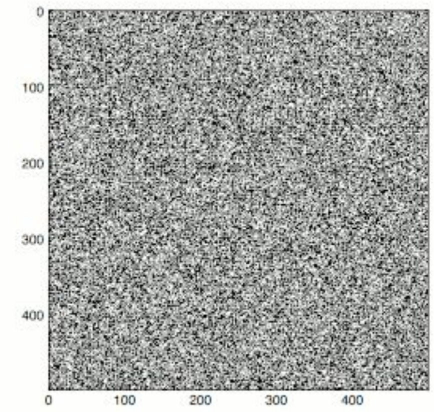
$$H = -J\sum_{i=1}^{N} s_i s_{i+1} - H\sum_{i=1}^{N} s_i$$

$$H = -\sum_{i=1}^{N}\left[J s_i s_{i+1} + H\frac{s_i + s_{i+1}}{2}\right]$$

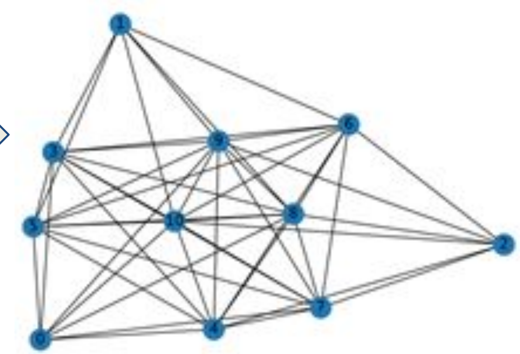
$$Z = e^{\Psi} = \sum_j e^{-\beta H(\mathbf{x}_j, \mathbf{p}_j)}$$

$$z(x) = \begin{cases} -1 & \text{if} \quad x = 0 \\ +1 & \text{if} \quad x = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Z = \sum^{1}\sum^{1}\cdots\sum^{1}\exp\left(\beta\sum^{N}\left[Jz(s_i)z(s_{i+1}) + H\frac{z(s_i)+z(s_{i+1})}{2}\right]\right)$$



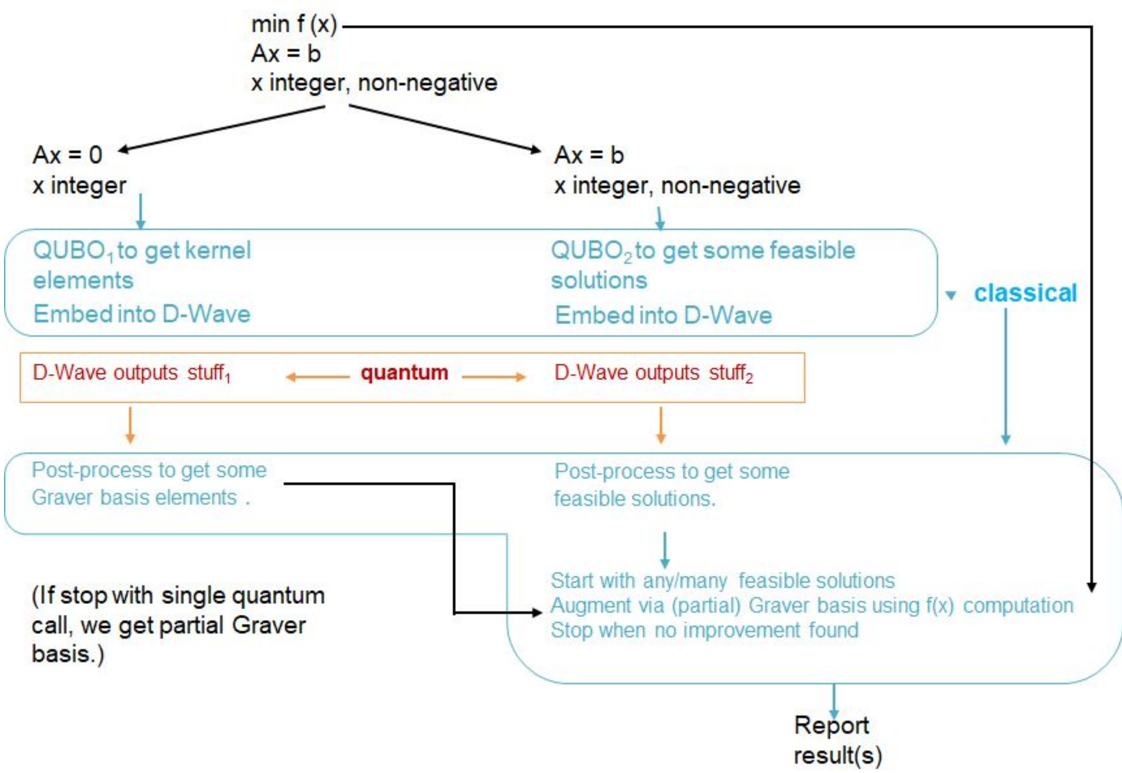
$$\begin{array}{c}\min cx \\ Ax = b \\ \mathbf{x} \in \{0,1\}^n\end{array} \Rightarrow \min_{\mathbf{x}} c\mathbf{x} + \rho(A\mathbf{x} - b)^T(A\mathbf{x} - b),\ \mathbf{x} \in \{0,1\}^n = \min_{\mathbf{x}} \mathbf{x}^T Q\mathbf{x} + c,\ \mathbf{x} \in \{0,1\}^n \Rightarrow$$

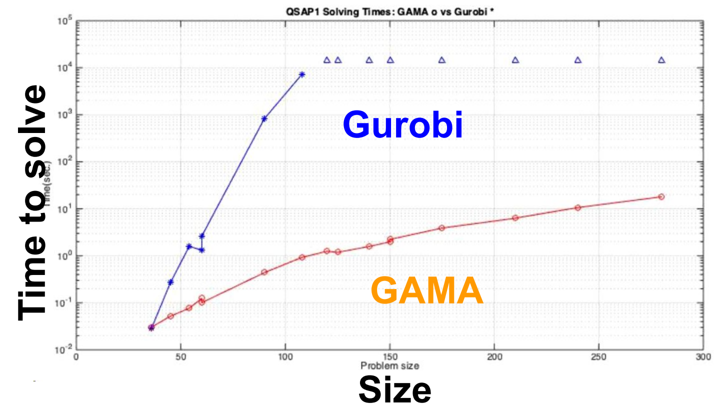


[1] https://en.wikipedia.org/wiki/Ising_model
[2] Algebraic And geometric ideas in the theory of discrete optimization. De Loera, Hemmecke, Köppe. 2012

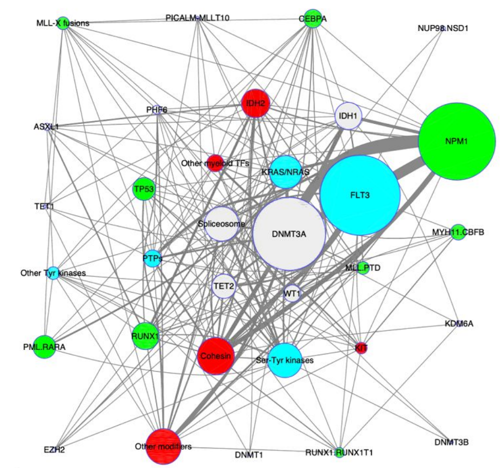
Graver Augmented Multiseed Algorithm - GAMA

Algorithm and applications





Quadratic Assignment Problem



Cancer Genomics

[1] https://arxiv.org/pdf/1902.04215.pdf
[2] https://arxiv.org/pdf/1907.10930.pdf
[3] https://www.biorxiv.org/content/10.1101/845719v1.full.pdf

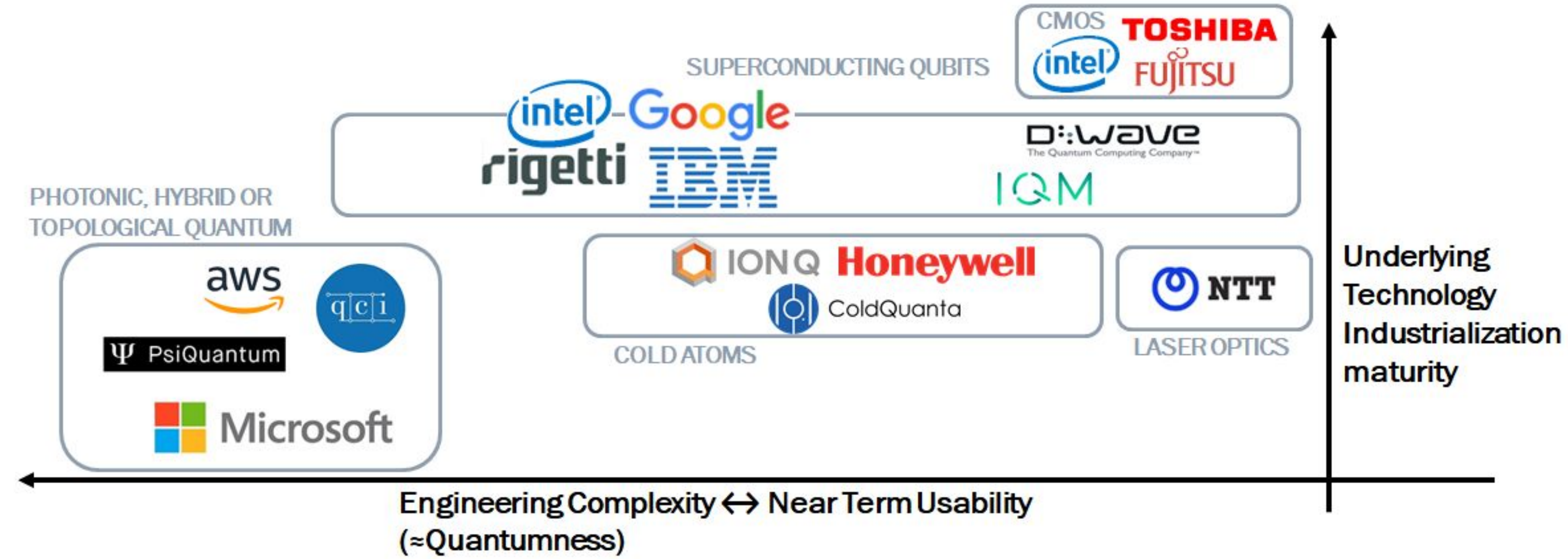
Unconventional Computing

Three Strategies, Multiple Technologies

FAULT-TOLERANT QUANTUM:
- Phase Estimation
- Amplitude Amplification/Estim.
- Sampling

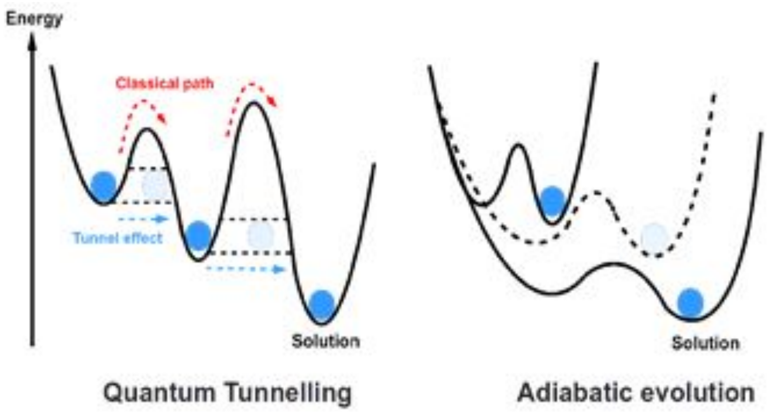
GATE-MODEL NISQ:
- Quantum Approximate Optimization
- Quantum Alternate Operator Ansatz
- Variational Quantum Eigensover
- Quantum Neural Networks

ANALOG:
- Quantum Annealing
- Coherent (Optical) Ising Machines
- Oscillator-based Computing
- Quantum-Inspired Digital Annealers

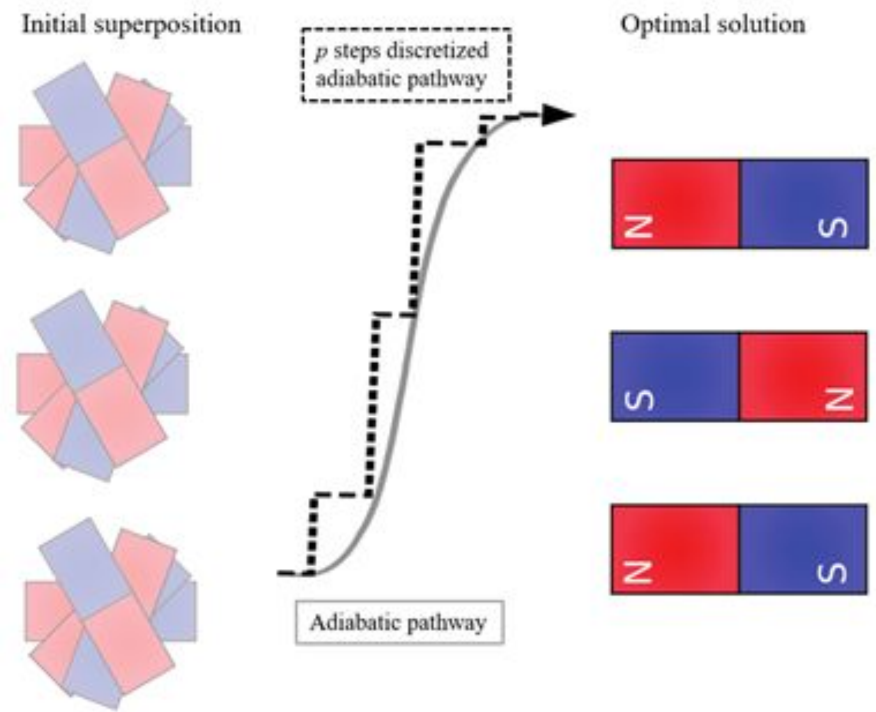


Quantum methods for solving Ising/QUBO

Adiabatic Quantum Computation



Quantum Annealing and QAOA



Gate-based computers and Quantum Annealers

[1]https://miro.medium.com/max/2420/1*n0wMIZVftp8cVLW8Mn6_Ew.png
[2]https://www.ibm.com/blogs/research/2017/11/the-future-is-quantum/
[3]https://www.dwavesys.com/press-releases/d-wave-makes-new-lower-noise-quantum-processor-available-leap

Recent Results: QA

Recent applied, advanced use (paused annealing, reverse annealing):

Ferromagnetically shifting the power of pausing

Zoe Gonzalez Izquierdo,[1,2,3] Shon Grabbe,[2] Stuart Hadfield,[2,3] Jeffrey Marshall,[2,3] Zhihui Wang,[2,3] and Eleanor Rieffel[2]

[1] *Department of Physics and Astronomy, and Center for Quantum Information Science & Technology, University of Southern California, Los Angeles, California 90089, USA*
[2] *QuAIL, NASA Ames Research Center, Moffett Field, California 94035, USA*
[3] *USRA Research Institute for Advanced Computer Science, Mountain View, California 94043, USA*
(Dated: June 16, 2020)

Leveraging Quantum Annealing for Large MIMO Processing in Centralized Radio Access Networks

Minsung Kim, Princeton University, minsungk@cs.princeton.edu
Davide Venturelli, USRA Research Institute for Advanced Computer Science, DVenturelli@usra.edu
Kyle Jamieson, Princeton Univeristy, kylej@cs.princeton.edu

Quantumness:

REPORT

Phase transitions in a programmable quantum spin glass simulator

R. Harris[1,*], Y. Sato[1], A. J. Berkley[1], M. Reis[1], F. Altomare[1], M. H. Amin[1,2], K. Boothby[1], P. Bunyk[1], C. Deng[1], ...
+ See all authors and affiliations

Science 13 Jul 2018:
Vol. 361, Issue 6398, pp. 162-165
DOI: 10.1126/science.aat2025

Letter | Published: 22 August 2018

Observation of topological phenomena in a programmable lattice of 1,800 qubits

Andrew D. King, Juan Carrasquilla, [...] Mohammad H. Amin

Nature **560**, 456–460(2018) | Cite this article

Benchmarking:

REPORT

Defining and detecting quantum speedup

Troels F. Rønnow[1], Zhihui Wang[2,3], Joshua Job[3,4], Sergio Boixo[5,6], Sergei V. Isakov[7], David Wecker[8], John M. Martinis[9], Dan...
+ See all authors and affiliations

Science 25 Jul 2014:
Vol. 345, Issue 6195, pp. 420-424
DOI: 10.1126/science.1252319

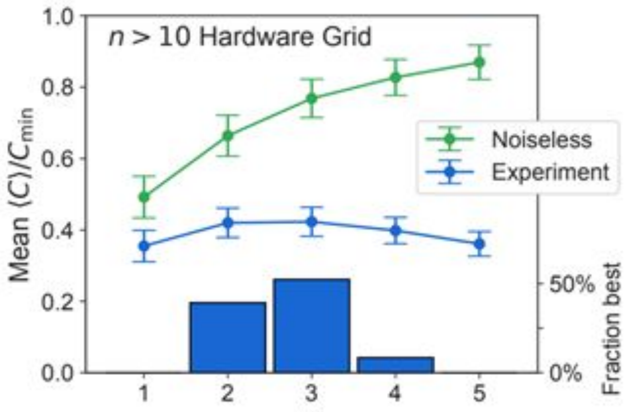
What is the Computational Value of Finite-Range Tunneling?

Vasil S. Denchev, Sergio Boixo, Sergei V. Isakov, Nan Ding, Ryan Babbush, Vadim Smelyanskiy, John Martinis, and Hartmut Neven
Phys. Rev. X **6**, 031015 – Published 1 August 2016

Recent Results: QAOA

Quantum Approximate Optimization of Non-Planar Graph Problems on a Planar Superconducting Processor

Google AI Quantum and Collaborators*
(Dated: April 10, 2020)



Reference	Date	Problem topology	$\Delta(G)$	n	p	Optimization
Otterbach *et al.* [22]	2017-12	Hardware	3	19	1	Yes
Qiang *et al.* [27]	2018-08	Hardware	1	2	1	No
Pagano *et al.* [26]	2019-06	Hardware[1] (system 1)	n	12, 20	1	Yes
		Hardware[1] (system 2)	n	20–40	1–2[(2)]	No
Willsch *et al.* [23]	2019-07	Hardware	3	8	1	No
Abrams *et al.* [24]	2019-12	Ring	2	4	1	No
		Fully-connected	n			No
Bengtsson *et al.* [25]	2019-12	Hardware	1	2	1, 2	Yes
This work		Hardware	4	2–23	1–5	Yes
		3-regular	3	4–22	1–3	Yes
		Fully-connected	n	3–17	1–3	Yes

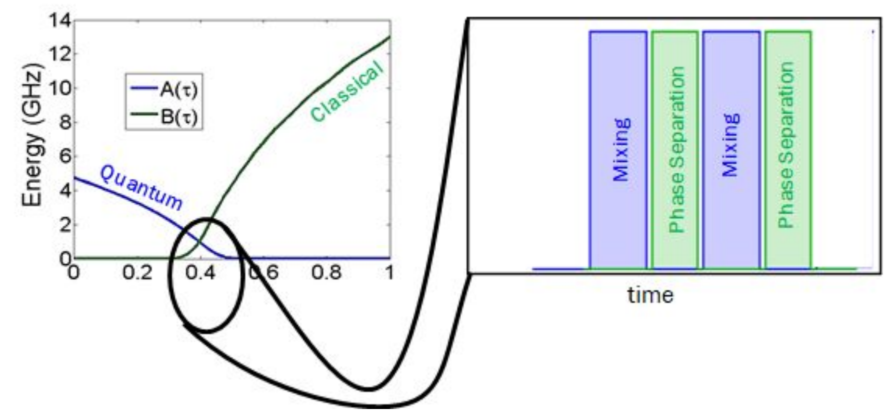
Optimizing Variational Quantum Algorithms Using Pontryagin's Minimum Principle

Zhi-Cheng Yang,[1] Armin Rahmani,[2,3] Alireza Shabani,[4] Hartmut Neven,[4] and Claudio Chamon[1]

Low depth mechanisms for quantum optimization

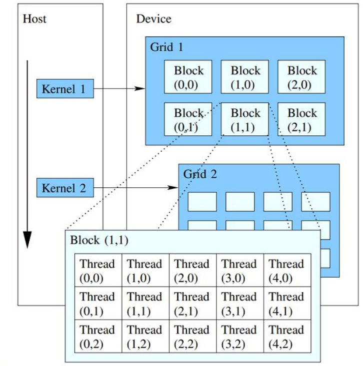
Jarrod R. McClean,[1,*] Matthew P. Harrigan,[1] Masoud Mohseni,[1] Nicholas C. Rubin,[1] Zhang Jiang,[1] Sergio Boixo,[1] Vadim N. Smelyanskiy,[1] Ryan Babbush,[1] and Hartmut Neven[1]
[1] *Google Research, 340 Main Street, Venice, CA 90291, USA*
(Dated: August 21, 2020)

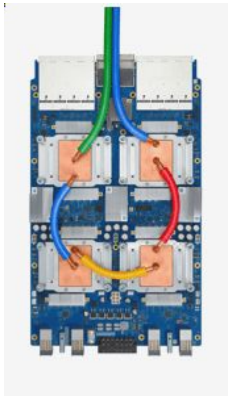
Relationship between QA to QAOA



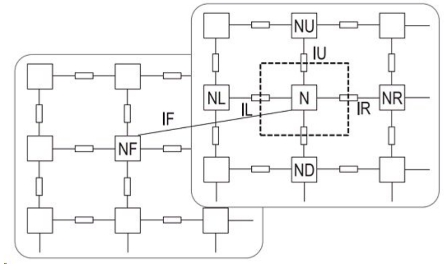
Specialized Hardware for solving Ising/QUBO

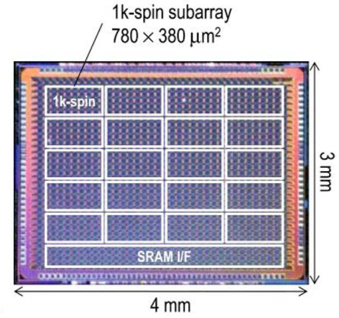
GPUs and TPUs





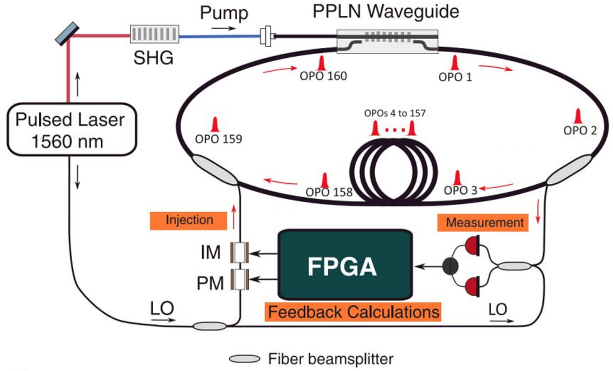
Complementary metal-oxide semiconductors (CMOS)





Digital annealers

Oscillator Based Computing

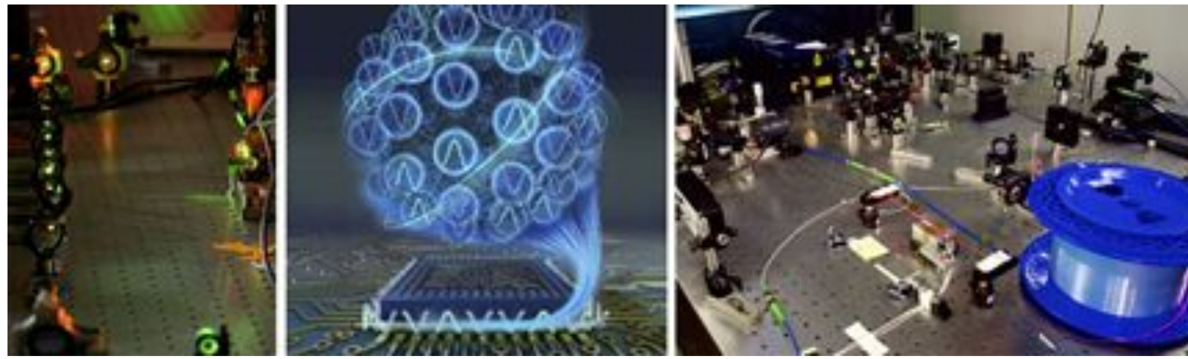


[1]https://arxiv.org/pdf/1807.10750.pc
[2]https://arxiv.org/pdf/1903.11714.pc
[3]https://arxiv.org/pdf/1806.08815.pc
[4]https://spectrum.ieee.org/tech-talk
mos-digital-annealer-produces-quantum-computer-speeds
[5]https://science.sciencemag.org/content/sci/354/6312/614.full.pdf

Guest Speaker Prof. P. McMahon (Cornell University)

https://mcmahon.aep.cornell.edu/index.html

FOCUS: EXPERIMENTAL AND THEORETICAL QUANTUM, PHOTONIC, AND NEUROMORPHIC COMPUTING



RESEARCH ARTICLE | APPLIED PHYSICS

Experimental investigation of performance differences between coherent Ising machines and a quantum annealer

Ryan Hamerly[1,2,*,†], Takahiro Inagaki[3,*,†], Peter L. McMahon[2,4,5,*,†], Davide Venturelli[6,7], Alireza Marandi[4,8], T...
+ See all authors and affiliations

Science Advances 24 May 2019:
Vol. 5, no. 5, eaau0823
DOI: 10.1126/sciadv.aau0823

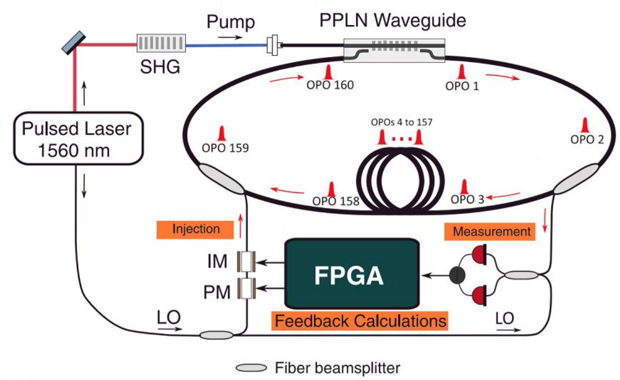
REPORT

A fully programmable 100-spin coherent Ising machine with all-to-all connections

Peter L. McMahon[1,2,*,†], Alireza Marandi[1,*,†], Yoshitaka Haribara[2,3,4], Ryan Hamerly[1], Carsten Langrock[1], Shuhei Tamate[2], T...
+ See all authors and affiliations

Science 04 Nov 2016:
Vol. 354, Issue 6312, pp. 614-617
DOI: 10.1126/science.aah5178

Coherent Ising Machines

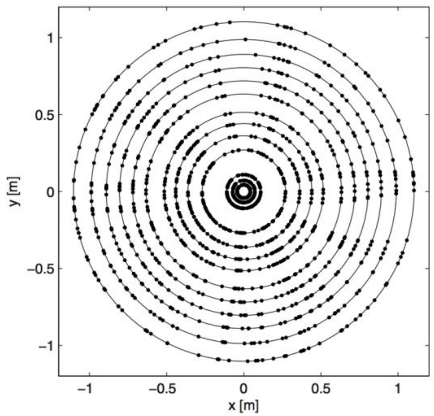


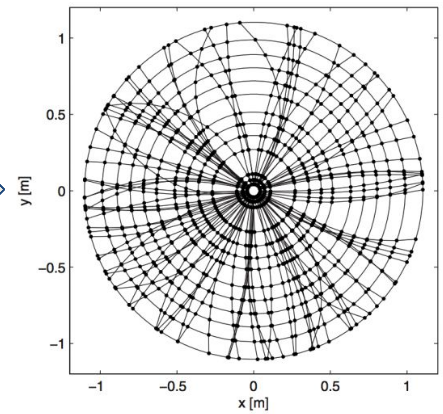
Grading Policy

- Weekly homework or quizzes (50%)
  - Each week will have a short quiz to evaluate concepts covered in previous lectures
  - Two worst quizzes won't be counted
- Scribes
  - Each student will be the scribe of one lecture, and this will count as a quiz
- Final Project (50%)
  - Group project (2-4 people). Weekly meeting with D. Bernal to check progress.
  - Formulate an important practical problem as a IP in multiple ways (formulations).
  - Solve at least one formulation via classical OR solvers (Gurobi or CPLEX).
  - Formulate the same practical problem for a quantum solver, performing resource estimation and solve a proof-of-concept instance on a real device or simulator.
  - Solve the same practical problem using GAMA (either GAMA-Q or GAMA-C).
- Deliverables:
  - Code to implement project
  - Final report highlighting knowledge acquired in the process
  - Final presentation in front of the whole class

Project proposal

Particle tracking in accelerators

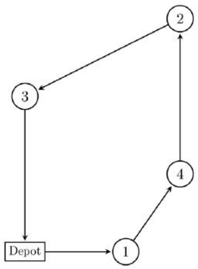


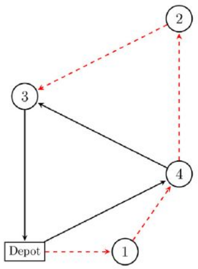


Measurement ⇨ Trajectories

https://arxiv.org/pdf/1908.04475.pdf

Drone-truck delivery problems





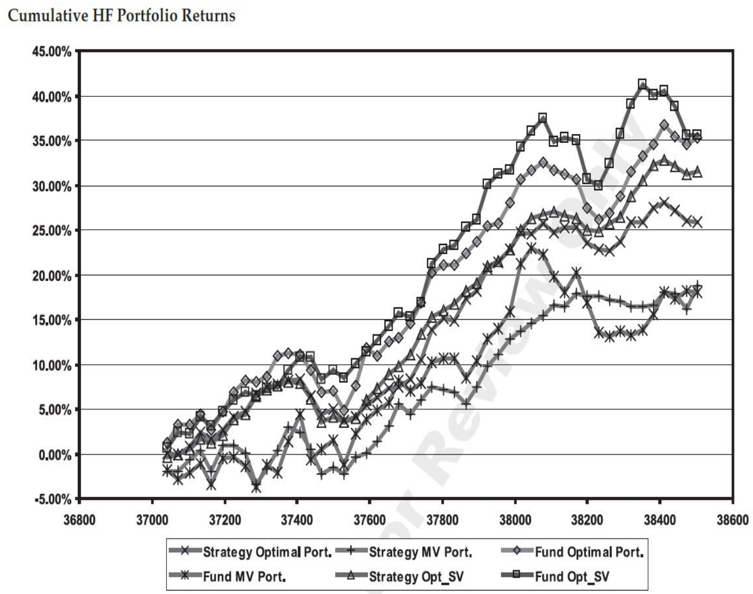
Drone-truck truck drone-truck

https://doi.org/10.1016/j.cor.2020.105004

- Other projects:
  - Bring your own application!
  - It has to be a combinatorial optimization problem of interest

Project proposal from Industry

Principal Financial Group: Hedge Fund portfolio optimization with Kurtosis and Skewness

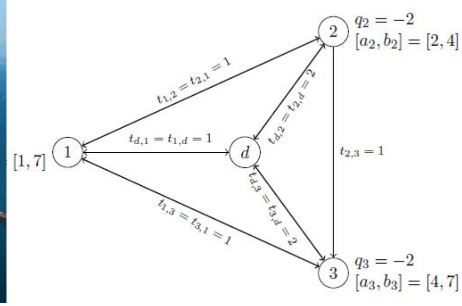


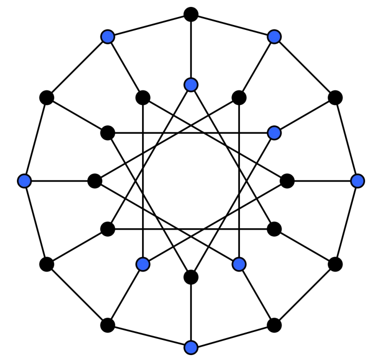
Purpose ***(What is the project motivation?)***	• The purpose of this project is to adopt Ising's model to solve a complex Optimization problem incorporating higher moments i.e. Skewness and Kurtosis into the objective function. • The goal is to establish a framework through which a reformulation of the problem is possible. • Its purpose is also to push the boundaries of optimization problem by incorporating other practical constraints such as mixed-integer formulation.
Objectives ***(What are we going to do?)***	• Perform a literature review of Ising's model with particular focus on its relevance to Portfolio Optimization • Reformulate the Optimization problem incorporating higher order moments so that it is amenable to be solved using the Ising's model. • Develop a working prototype of the proposed methodology • Define metrics and validation procedures for evaluating model performance – both accuracy as well speed. • Define and prototype benchmarks for comparison. Consider baseline Mean Variance Optimization for this purpose. • Demonstrate a prototype of the developed methodology on a large dataset
Output ***(What are the project deliverables?)***	• A thorough report of literature review identifying literature relevant to adopting Ising's model to Portfolio Optimization. • Establish a framework for reformulating the optimization problem using Ising's model. • A working prototype implementing Ising's model for addressing complex Portfolio Optimization problem. • Documentation for the proposed mathematical formulation.
Outcome ***(Expected impact on which sub-system(s)?)***	• A richer formulation incorporating higher order moments of returns namely, Skewness and Kurtosis in addition to Mean and Variance leads to renewed focus on traditionally neglected aspects of return distribution. • This reformulation would play a pivotal role in transforming the portfolio manager mindset and development of this framework would allow us to rapidly prototype and readily implement it. • Incorporating higher order moments of return is beset with mathematical complexities and obstacles arising from practical computational limitations. This novel approach to solve this problem circumvents these traditional limitations.

Project examples

Maritime Routing Problem

Stable-set of a graph





Real life application

Graph Theoretical problem

- Other applications in Finance, Engineering, and Sciences

Other applications

Applied:

- Air Traffic Management
- Portfolio Optimization
- Airport Gate Scheduling
- Autoencoders
- Anomaly detection in networks
- Vehicle Routing
- Robot Operations Planning

Paradigmatic:

- SAT
- Traveling Salesman Problem
- Job Shop Scheduling
- Spin Glasses

Course Policy

This is the first time this lecture is being taught and the topic is novel so the aim is to learn a lot and have fun!

- Auditing students are encouraged to participate actively in the lectures
  - Consider doing the project, one learns by doing
- Regular attendance is essential and expected
- CMU students: use canvas
  - The quizzes are being posted there
  - Questions should be asked there to make it available to everyone

- Academic honesty is expected. Refer to the CMU's policies on academic integrity when in doubt.

Videos and extra resources

- Course Website
  - https://bernalde.github.io/QuIP/
- Teaser video
  - https://www.linkedin.com/posts/carnegie-mellon-tepper-school-of-business_quantum-computing-activity-6698655542186913792-001_
- CMU Quantum Computing Group Website
  - https://lnkd.in/d6m5ECV
- USRA Computer Science and Information Technology
  - https://riacs.usra.edu/quantum (includes a full login-protected QC course)
- Pittsburgh Quantum Institute
  - https://www.pqi.org/
- NASA Quantum and Artificial Intelligence Laboratory (QuAIL)
  - https://quantum.nasa.gov
- Prof. Tayur's seminar at Cornell on GAMA
  - https://cornell.hosted.panopto.com/Panopto/Pages/Viewer.aspx?id=3d46643f-03ea-4e3f-ad7a-ab9901290472

USRA collaboration

- Development of this course was supported by the Air Force Research Lab NYSTEC-USRA Contract (FA8750-19-3-6101)

- Access to D-Wave systems might be available via written proposals to the University Space Research Association (USRA). See https://riacs.usra.edu/quantum/rfp for terms and conditions. The course will discuss proposal preparation.

- Students of this course are encouraged to apply to the Feynman Academy Internship program https://riacs.usra.edu/quantum/qacademy that sponsors research projects at NASA Ames Research Center.

Why Universities Exist

"The justification for a university is that it preserves the connection between knowledge and zest of life, by uniting the young and old in the imaginative consideration of learning…The task of the university is to weld together imagination and experience…..The task of the university is the creation of the future…."